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Oscillatory Burning of High-Pressure Exponent Double-Base Propellants

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Nomenclature

a = coefficient in Vieilles burning rate law
C_d = nozzle discharge coefficient
K_n = ratio of propellant burning area to nozzle throat area
L^* = characteristic chamber length
n = pressure exponent in Vieilles burning rate law
p = pressure
p_c = mean chamber pressure
R = gas constant
r = linear burning rate
T_0 = propellant conditioning temperature
T_f = flame temperature
t = time
α = exponential growth constant of oscillation
α_t = thermal diffusivity of propellant
ρ_p = propellant density
τ = lead time of burning rate oscillation relative to pressure oscillation
τ_{ch} = chamber time constant, L^*/C_dRT_f
τ_{th} = thermal wave relaxation time, α_t/r^2
ϕ = amplitude of pressure oscillation
ω = frequency

Introduction

THE most familiar examples of low-frequency oscillatory burning are associated with chuffing experienced with double-base propellants at low pressures. Many of the experimental studies of chuffing and low-frequency burning have been conducted, and theoretical models have been presented to describe the burning behavior.[1-3] The models have been moderately successful; however, they use uncertain values for parameters such as the burning surface temperature and the activation energy of the burning surface.

In this Note, very-low-frequency burning of high-pressure exponent propellants is investigated using a strand burner and an L^* burner. The oscillatory burning was observed at intermediate pressures and was found to be different from the oscillatory burning of chuffing. The domain of the oscillatory burning (L^* vs p) is parabolic. An analytic model predicting the domain of oscillatory burning and the characteristics of oscillation in the case of very-low-frequency burning are presented.

Experiments

The propellant used in this study was a double-base propellant (51.0% nitrocellulose, 35.5% nitroglycerine, 12.0% diethyl phthalate, and 1.5% lead salicylate). The burning rate measurements were carried out with a nitrogen-pressurized chimney-type strand burner. Tests for unstable burning domains were done with a small end-burning-type rocket motor (L^* burner). Propellant grains were 114 mm diam × 40 mm long with the side and one end of the grain coated by an inhibitor. The burning pressure was measured by a strain-guage type pressure transducer mounted to the motor and was recorded by a visicorder. Since the chamber free volume increased with regressing the burning surface of the propellant grain, the L^* increased with burning time. The burning tests were conducted with L^* = 4-20 m.

The burning rate and pressure exponent of the propellant are largely dependent on pressure range. The burning rate characteristics are divided into four pressure zones, as designated in Fig. 1. The pressure exponent is 0.44 in zone I above 37 atm, approximately 1.1 in zone II between 37 and 21 atm, 0.77 in zone III between 21 and 11 atm, and 1.4 in zone IV below 11 atm.

The burning test results by the L^* burner showed that the burning behavior was very dependent on the pressure range and L^*. Typical pressure-time records are shown in Fig. 2. In zone I pressures above 37 atm, the burning was very stable, and neither oscillatory burning nor irregular burning was observed. In zone II where the pressure exponent was approximately unity, a sinusoidal oscillatory burning was observed when L^* was short. However, the oscillation diminished as L^* increased during burning. L^* observed oscillatory burning ranged 4-10 m, and the highest amplitude of pressure oscillation was approximately 7 atm. The frequency increased with increasing pressure from 6 Hz at 21 atm to 8

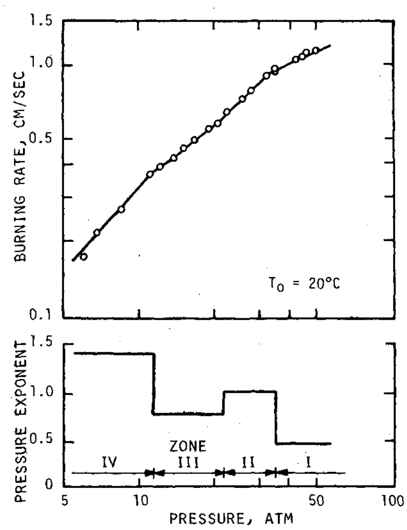

Fig. 1 Burning rate and pressure exponent data showing two high-pressure exponent zones.

Presented as Paper 76-668 at the AIAA/SAE 12th Propulsion Conference, Palo Alto, Calif., July 26-29, 1976; submitted Aug 16, 1976; revision received Oct. 22, 1976.

Index category: Combustion Stability, Ignition, and Detonation.

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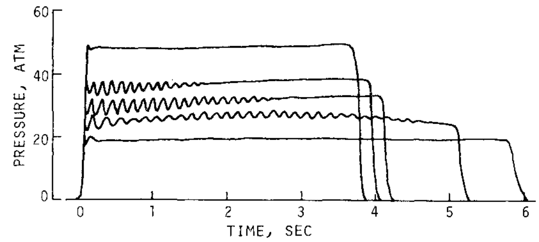

Fig. 2 Typical pressure-time records of L^* burner tests showing that very-low-frequency burning occurs only in the limited-pressure range.

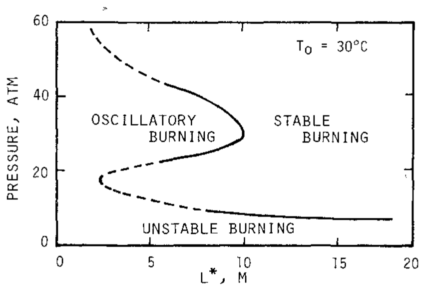

Fig. 3 Domain of oscillatory burning that appeared to be parabolic in shape.

Hz at 37 atm. The amplitude of oscillation decreased with increasing L^* whereas the frequency did not change.

On the other hand, stable burning without oscillatory burning was observed in zone II where the pressure exponent was 0.77, pressure between 21 and 11 atm. The burning was stable even at the shortest length of L^* (4 m) used in this experiment. In zone IV, stable burning was not possible below 11 atm. The propellant grains were consumed by the fizz burning at atmospheric pressure. Thus, the domain of the observed low-frequency burning appeared to be parabolic as shown in Fig. 3.

Analysis

Experimental results showed that the oscillatory burning observed at intermediate pressure in this experiment is different from the oscillatory burning associated with chuffing at low pressures. The following analysis, which is based on the concept presented by Beckstead and Price,[4] may be applied for the elucidation of the very-low-frequency oscillatory burning observed in zone II.

Assuming the ideal gas in the chamber and neglecting the density of the gas compared to that of the propellant, the mass balance for a rocket motor is given by

$$\mathrm{d}p/\mathrm{d}t + (C_d R T_f / L^*)p - (\rho_p K_n R T_f / L^*) r = 0 \tag{1}$$

It is assumed that the amplitude of the pressure oscillation is sinusoidal as

$$p(t) = p_c + \phi e^{\alpha t} \cos\omega t \tag{2}$$

In the case of the oscillation being very slow, the period of oscillation, $2\pi/\omega$, is much larger than the characteristic time of the thermal wave of the propellant τ_{th}, thus a linear burning rate law $r = ap^n$ is possible to apply during oscillating burning. Assuming that the propellant burns with lead time τ relative to the pressure oscillation, the instantaneous burning rate is

$$r(t) = a\{p(t+\tau)\}^n = a\{p_c + \phi e^{\alpha(t+\tau)} \cos\omega(t+\tau)\}^n \tag{3}$$

In general, the amplitude of the pressure oscillation is much smaller than the mean pressure as $\phi/p_c << 1$, thus the burning rate is given by

$$r(t) = ap_c^n + anp_c^{n-1} \phi e^{\alpha(t+\tau)} \cos\omega(t+\tau) \tag{4}$$

Substituting Eqs. (2) and (4) into Eq. (1), the following set of equations is obtained[4]

$$\cos\omega\tau = (1 + \alpha\tau_{ch})/n \tag{5a}$$

$$\sin\omega\tau = \omega\tau_{ch}/n \tag{5b}$$

Combining Eqs. (5a) and (5b), we get the following relationship between α and ω

$$(1 + \alpha\tau_{ch})^2 + (\omega\tau_{ch})^2 = n^2 \tag{6}$$

Since the pressure exponent of the burning rate is less than unity for conventional propellants, α becomes negative, and the burning becomes stable. In the case of n being greater than unity, α becomes positive, and a growing oscillatory burning may occur. When n is very close to unity, α becomes approximately zero, and ω can be determined by the approximation

$$\omega \simeq (n^2 - 1)^{1/2}/\tau_{ch} \tag{7}$$

Discussion

In the analysis presented in this study, α and ω are not uniquely determined at given τ_{ch} and p_c. However, Eq. (5a) gives the relationship

$$\alpha \leq (n-1)/\tau_{ch} \tag{8}$$

The above unequality shows that α decreases with increasing τ_{ch} at given pressure. In other words, oscillatory burning diminishes with increasing L^*. This predicted trend agrees with the observed oscillatory burning behavior in zone II. Since the α in zone II is determined to be on the order of 1/sec or less, ω is calculated from Eq. (6). For example, the calculated ω is 46 rad/sec when τ_{ch} is 0.01 sec ($L^* \simeq 4$ m), which is very close to the observed value 44 rad/sec at the mean pressure 35 atm. Furthermore, the term $\omega\tau_{ch}/n$ is the order of 0.1 rad in the oscillatory zone, it is obtained from the approximation of Eq. (5b) $\omega\tau_{ch}/n \simeq \omega\tau$. Thus, τ is determined to be equal to τ_{ch}.

In conclusion, the model presented in this study is not complete to understand the detailed mechanism of oscillatory burning; however, it provides an overall view of the observed very-low-frequency burning. It was found that the lead time of burning rate oscillation is approximately equal to the chamber time constant when the pressure exponent is about unity.

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